Grade 6 Mathematics, Quarter 3, Unit 3.1

Read, Write, and Evaluate Expressions Through Formulas

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Fluently add and subtract multidigit decimals using the standard algorithm.
- Fluently multiply multidigit decimals using the standard algorithm.
- Fluently divide multidigit decimals using the standard algorithm.
- Find the volume of a right rectangular prism with fractional edge lengths.
- Apply the formula of volume to a right rectangular prism.
- Evaluate expressions involving variables.
- Evaluate expressions from formulas used in real-world problems.
- Perform mathematical operations involving whole number exponents.

Mathematical practices to be integrated

Model with mathematics.

- Use algorithms to add, subtract, multiply, and divide multidigit decimals.
- Use the formula to find volume of right rectangular prisms.
- Use algorithms for operations involving exponents.

Attend to precision.

- Read, write, and evaluate expressions in which letters stand for numbers.

Look for and express regularity in repeated reasoning.

- Evaluate the reasonableness of their results.

Essential questions

- Which strategies are helpful when adding/subtracting multidigit decimals?
- Which strategies are helpful when multiplying and dividing multidigit decimals?
- How do you find the volume of a right rectangular prism with fractional edge lengths?
- How would you use the formula to find the volume of a right rectangular prism?
- What do the variables for volume represent?
- How can I use an algebraic expression to model a real world situation?
- Why is it important to use the order of operations in solving expressions with exponents?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System 6.NS

Compute fluently with multi-digit numbers and find common factors and multiples.

6. NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

6. G.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Expressions and Equations 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

6. EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\ s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of

the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 4, students fluently add and subtract multidigit numbers using the standard algorithms.

Students in grade 5 learn about place value and fluently multiply multidigit numbers using the standard algorithms. They develop an understanding of volume.

Earlier in the sixth grade, students had to find the area of right triangles, other triangles special quadrilaterals, and other polygons by composing into rectangles or decomposing into triangles and/or other shapes; they apply these techniques in the context of solving real-world and mathematical problems.

Current Learning

By the end of sixth grade, students should be fluently dividing multidigit numbers using the standard algorithms.

Also by the end of grade 6, students must find the volume of rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism.

This is the first time students are introduced to variables in which letters stand for numbers and to write expressions. Students will identify parts of an expression using mathematical terms. Finally, students will evaluate expressions that arise from formulas used in the real world.

Future Learning

In seventh grade, they will convert a rational number to a decimal using the long division, and must know that the decimal form of a rational number terminates in zeros or eventually repeats.

They will to solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, other polygons, cubes and right prisms.

They will use properties of operations to generate equivalent expressions. They will fluently be able to solve real-life and mathematical problems using numerical and algebraic expressions and equations. Students will solve multistep, real-world mathematical problems with positive and negative numbers. Students will use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Additional Findings

In the *PARCC Progressions 6–8 Expressions and Equations*, the author states, "It is important to distinguish between such conventions, which are notational conveniences that allow for algebraic expressions to be written with fewer parentheses, and properties of operations, which are fundamental properties of the number system and undergird all work with expressions."(p. 6)

According to *Principles and Standards for School Mathematics*: "the measurement and geometry standards span several important middle-grade topics, such as similarity, area, perimeter, area, volume, and classification of shapes that depend on side lengths or angle measures." (p. 241)

In the *PARCC Progressions 6–8 Geography*, the author states: "Students' competencies in shape composition and decomposition especially with spatial structuring of rectangular arrays (recall p. 11) should be highly developed. These competencies form a foundation for understanding multiplication, formulas for area and volume, and the coordinate plane." (p. 18)

Grade 6 Mathematics, Quarter 3, Unit 3.2

Using a Number Line to Understand Rational Numbers

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand that positive and negative integers describe quantities that have opposite directions or values.
- Understand that the negative symbol indicates the opposite of any integer.
- Locate a negative integer on the number line.
- Use positive and negative integers to represent quantities in real-world contexts.
- Explain the meaning of zero within real-world situations.
- Recognize opposite signs of integers as indicating locations on opposite sides of zero on the number line.
- Recognize opposite integers are the same distance away from zero on opposite sides of the number line.
- Recognize that zero is its own opposite.
- Write, interpret, and explain statements of order for rational numbers in real-world contexts.
- Understand the absolute value of a rational number as its distance from zero on the number line.
- Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- Distinguish comparisons of absolute value from statements about order.

Mathematical practices to be integrated

Attend to precision.

- Label and place integers on a number line accurately.
- Locate the opposite signs of integers on a number line.
- Explain statements for rational numbers in real world contexts.

Look for and express regularity in repeated reasoning.

- Recognize that opposites are always the same distance from zero.
- Recognize that absolute value is always positive.

Essential questions

- How do you locate an integer on a number line?
- How do you identify an integer on a number line?
- What is the relationship of zero to the opposite integers?
- What is the meaning of the negative symbol?
- What is a real-world situation that uses both positive and negative numbers including zero?
- What is the meaning of zero?
- What do you know about numbers and their opposites?
- What is a rational number?
- How do you locate a rational number on a number line?
- What do you know about numbers and their position on a number line?
- Where is the idea of absolute value used outside of the math classroom?
- How do you write an inequality of two rational numbers on a number line?
- How do inequality statements relate to the real-world problems?
- How are order (inequality) statements different from absolute value statements?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System	**6.NS**

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself,
e.g., –(–3) = 3, and that 0 is its own opposite.

6. NS.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3 ºC > –7 ºC to express the fact that –3 ºC is warmer than –7 ºC.*

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.*

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.*

Common Core State Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

Throughout the elementary years, students used the number line to understand positive numbers. Students use number line models to compare, add, subtract, multiply, and divide whole numbers and fractional quantities.

Current Learning

This will be the students' introduction to the concept of negative integers and their placement of the number line. It is important to connect the students' prior knowledge of real-world situations that include both positive and negative numbers. Students will extend their understanding of positive numbers to include negative numbers. They will generalize their knowledge of integers to include their location on a number line in relation to zero. Students will understand ordering and absolute value of rational numbers.

Future Learning

In grade 7, the students will apply this understanding to include operations with negative numbers. Students in future grades are expected to apply the concept of rational numbers to other content and standards. Rational numbers are not directly addressed in future grades.

Additional Findings

According to *Principles and Standards for School Mathematics,* "students should develop meanings for integers and represent and compare quantities with them" (p. 393).

In the *PARCC Progressions* for sixth grade, the author states: "Students must be able to place rational numbers on a number line before they can place ordered pairs of rational numbers on a coordinate plane. The former standard about ordering rational numbers is much more fundamental" (p. 2).

Grade 6 Mathematics, Quarter 3, Unit 3.3

Understanding Rational Numbers in a Coordinate Plane

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Understand rational numbers on a number line.
- Extend number line knowledge to points on a coordinate plane with negative number coordinates.
- Understand signs of numbers in an ordered pair by locating in a quadrant of the coordinate plane.
- Recognize ordered pairs with different signs are reflections across one or both axes.
- Find and position integers and rational numbers on horizontal or vertical number lines.
- Find and position any ordered pair on a coordinate plane and apply it to real-world problems.
- Find the distance between two points in the coordinate plane with the same first or second coordinate.
- Solve real-world mathematical problems by graphing in all four quadrants of the coordinate plane.
- Use absolute value to find coordinate distances between two points.
- Draw polygons on a coordinate plane and find their side lengths in real-world situations.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Comparing the methods students used to plot points on a number line and coordinate planes.
- Communicate accurately how to plot a rational number on a number line.

Model with mathematics.

- Draw polygons on a coordinate plane.
- Recognize ordered pairs with different signs are reflections across one or both axes.

Attend to precision.

- Label and place integers on a number line or coordinate plane accurately.
- Locate the opposite signs of integers on a coordinate plane.
- Explain statements for coordinate planes in real-world contexts.

Essential questions

- What is a rational numbers and how do you show it on a number line?
- How do you show negative numbers on a coordinate plane?
- How do you determine the point and quadrant on the coordinate plane using the sign?
- What points are reflections on a coordinate plane?
- How do you find and position integers and rational numbers on horizontal or vertical number lines?
- How do you find and position any ordered pair on a coordinate plane and apply it to real-world problems?
- How do you find the distance between two points in the coordinate plane with the same first or second coordinate?
- When, in the real world, would you graph in all four quadrants of the coordinate plane?
- How do you use absolute value to find coordinate distances between two points?
- When, in the real world, would you draw polygons on a coordinate plane and find their side lengths?

Written Curriculum

Common Core State Standards for Mathematical Content

The Number System	**6.NS**

Apply and extend previous understandings of numbers to the system of rational numbers.

6. NS.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

6. NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Geometry	**6.G**

Solve real-world and mathematical problems involving area, surface area, and volume.

6. G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Common Core State Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In previous grades, students used number lines to plot fractions and decimals. In fifth grade, students solved problems by graphing points in the first quadrant of the coordinate plane.

Current Learning

Students are extending their knowledge of the number line to include negative integers and other rational numbers on a number line. They extend their graphing ability from quadrant 1 (positive by positive coordinate pairs) to all four quadrants to include negative integers. Students are preparing for work on the scale drawings and instructions in grade 7 by drawing polygons in the coordinate plane.

Future Learning

In seventh grade, students will solve problems involving scale drawings and informal geometric constructions, and working with two-and three-dimensional shapes to solve problems involving area, surface area, and volume. They will be graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Additional Findings

In the *PARCC Progressions–Ratio and Proportional Relationships*: "Students graph the pairs of values displayed in ratio tables on coordinate axes. The graph of such a collection of equivalent ratios lies on a line through the origin, and the pattern of increases in the table can be seen in the graph as coordinated horizontal and vertical increase." (p. 5)

According to *Adding it up*; helping children learn mathematics: "The links from number to geometry and to algebra are forged even more strongly when students are introduced to the coordinate plane, in which perpendicular number lines provide a system of coordinates for each point." (p. 107)